（１）仮想仕事式

・静的可容応力場

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i = 0 \quad \text{in} \quad B_t$$

$$\sigma_{ij} n_j = t_i^0 \quad \text{on} \quad S_\sigma$$

・運動学的可容場（滑らかなベクトル場）

$$u_i = 0 \quad \text{on} \quad S_u$$

$$\int_{B_t} \left(\frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i \right) u_i \, dv = 0 \quad \rightarrow \quad \int_{B_t} \sigma_{ij} \varepsilon_{ij} dv = \int_{S_\sigma} t_i^0 u_i \, ds + \int_{B_t} \rho b_i u_i dv$$

$$\because \sigma_{ij} \frac{\partial u_i}{\partial x_j} = \sigma_{ij} \left[\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \right] = \sigma_{ij} \left[\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] = \sigma_{ij} \varepsilon_{ij}$$

（２）輸送定理

$$\frac{D}{Dt}\int_{B_t}\phi(\boldsymbol{x})dv \quad \rightarrow \quad \int_{B_t}\left(\frac{D\phi(\boldsymbol{x})}{Dt}+\phi\frac{\partial v_i}{\partial x_i}\right)dv=\int_{B_t}\left(\dot{\phi}+\phi\,\mathrm{div}\boldsymbol{v}\right)dv$$

$$\because \boldsymbol{x}\rightarrow\boldsymbol{X},\quad B_t\rightarrow B_0,\quad dv\rightarrow JdV:\quad J=\det F=\left|\frac{\partial x_i}{\partial X_j}\right|,\quad \frac{DJ}{Dt}=J\frac{\partial v_i}{\partial x_i}=J\ \mathrm{div}\boldsymbol{v}$$

（３）質量保存則および有用公式

$$\frac{D}{Dt}\int_{B_t}\rho(\boldsymbol{x})\,dv = 0 \quad \to \quad \frac{D\rho}{Dt}+\rho\frac{\partial v_i}{\partial x_i}=0,\ \frac{\partial\rho}{\partial t}+\frac{\partial(\rho v_i)}{\partial x_i}=0$$

$$\to \quad \dot{\rho}+\rho\,\mathrm{div}\boldsymbol{v}=0,\ \frac{\partial\rho}{\partial t}+\nabla(\rho\boldsymbol{v})=0$$

$$\frac{D}{Dt}\int_{B_t}\rho\phi\,dv=\int_{B_t}\rho\frac{D\phi}{Dt}dv=\int_{B_t}\rho\dot{\phi}\,dv$$

（４）運動方程式と静的つり合い式

$$\frac{D}{Dt}\int_{B_t} \rho v_i \, dv = \int_S t_i \, ds + + \int_{B_t} \rho b_i \, dv \quad \rightarrow \quad \rho \frac{Dv_i}{Dt} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i$$

（５）エネルギー保存則

$$\frac{D}{Dt}\int_{B_t}\rho\left(e+\frac{v_i v_i}{2}\right)dv=\dot{W}+\dot{Q}\quad\rightarrow\quad\rho\dot{e}=\sigma_{ij}\varepsilon_{ij}+\rho h-\frac{\partial q_i}{\partial x_i}=\boldsymbol{\sigma}:\boldsymbol{\varepsilon}+\rho h-\nabla\cdot\boldsymbol{q}$$

$$\because\dot{W}=\int_S t_i\,v_i\,ds+\int_{B_t}\rho b_i v_i dv,\quad\dot{Q}=-\int_S q_i\,n_i\,ds+\int_{B_t}\rho h\,dv$$